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TEACHING PUPILS HOW TO STUDY MATHEMATICS

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I

Of all the considerations connected with the study of mathematics, and indeed with the study of any subject, the most important is the mastery of the art of study itself. No topic has been so generally overlooked and neglected heretofore. This neglect is the source of many of our difficulties in teaching, and of many of the criticisms that have been heaped upon the various studies. A supervisor of the high schools of one of our states recently made a general outline for the reorganization of the courses in mathematics for the state. The matter of teaching pupils how to study had been entirely overlooked. His attention was called to the matter, and, realizing its importance, he made it a part of the program. After all, this is the chief thing to be gained from our schools. The pupil must learn the "how." The "what" is not of so great importance. The "what" frequently changes. The "how" is relatively constant. In other words, the pupil should learn how to attack a problem with economy of time and effort, and with the greatest efficiency. The information he gains in the process is incidental, and illustrative of what he ought to expect as a result of his efforts after he has been trained. It is this sort of training that gives the educated man a measure of his powers, and ability to use these powers in the most effective manner in the various problems which he meets in his daily living. If education fails in this it fails utterly; indeed, it is then not education at all, it is a farce, and the school is a failure. Yet this is the point at which the schools do fail most lamentably. No subject in the high school curriculum is equal to mathematics in its opportunities for teaching the art of study. Geometry is especially valuable for this purpose.

If the pupil is to realize the values to be obtained from the study of mathematics, it will usually be necessary that he master more completely the art of study, than he has done in the pursuit of his other studies. Indeed the ability to master mathematics means the mastery of this art, while failure in mathematics means inability to study properly. Heretofore it has been assumed that anyone who is willing to work hard can master the technique of study—that if teacher and parents, by combining their efforts, and with coaxing, urging, or threatening, can secure application and effort on the part of the pupil, all other desirable results must, in some unexplained and unexplainable way, follow. Nothing could be more seriously in error. The pupil with his book before him reads, reads, reads; and then, often with a frantic and usually futile effort, he tries to remember what he has read. He usually fails in this, but if he should succeed with the memory work and give what is considered a good recitation, there is little or no real gain to the pupil. In case of failure the teacher usually makes additional suggestions, such as reading aloud, or writing to aid the memory. Little, if any, reference is made to the fact that *thinking* is real study, and that one may read and recite indefinitely without doing any studying. Indeed, the things of secondary importance—learning and reciting facts—are emphasized so much, as to exclude the possibility of real study. While we assume that the pupil needs training for his vocation, this most important of all arts is left for him to learn without assistance. This is unfair to the pupil since it robs him of his rightful heritage—the right to benefit from the experience of others as far as is possible. Is it any wonder that so many pupils fail in school; or, at best, make an indifferent success of their work? Is it any wonder that mathematics, in which it is so absolutely necessary that one *study* to make real progress, or to get real enjoyment from it, is a failure, and that so much adverse criticism is heaped upon it? Teaching pupils how to study is the first and most important duty of the teacher.

One of the most serious difficulties lies in the fact that teachers themselves do not know, and hence, cannot teach pupils how to study. Dr. Lida B. Earhart, of Columbia University (in "*Teaching Children to Study*"), in connection with an experiment in graded schools to ascertain whether children were taught how to study, says:

"Teachers themselves are lacking the proper conception of the process of higher study; they tend to exalt memorizing; and they do not, as a class, accord recognition to any factor or factors as being essential to study. In several instances, the factors which they have recognized were employed largely by the pupils in their studies; and the factors which the teachers have overlooked in their resports were used but little by the pupils in their tests. . . . Although pupils possess ability to employ the various factors of proper study, the teachers lack a clear conception of what such study is. The teachers who wrote the questionnaires do not themselves employ these factors to any great extent; and the teachers observed in the classrooms are not training their pupils to use them. The teacher is the center and the moving power in nearly all of the work, and the requirements laid upon the pupils involve mechanical effort to a large degree. The aim of the work as a whole seems to be the mechanical acquisition of subject-matter.

"The development of the power to work independently, in telligently, and economically is almost entirely ignored. The teachers do not know of what such study consists and consequently give little thought to its cultivation. They would probably do so if they had definite ideas as to its nature, for they are frequently heard to lament the fact that their pupils do not know how to study or to think. Unfortunately the books on method give little or nothing in regard to the method of study. They deal almost exclusively with the teaching side of the school room situation, and do not say anything at all about training pupils to study, or else what they do contain is stated in such general terms that it benefits the teacher very little."

The teaching of pupils how to study should be made a necessary and an important part of the preparation of every teacher. However, the time to begin to teach the art of study, for both teachers and pupils, is at the beginning of their work in the grades.

Can pupils be taught to study? Do the difficulties connected with its mastery put it beyond the reach of elementary, and even of high school pupils? Dr. Earhart, on the basis of her experiment with fourth grade pupils, concludes that:

"Pupils in the elementary schools in grades including the fourth, as well as higher classes, are able, not only to employ the factors of logical study, but also by means of systematic effort they can be made to improve in their employment of them."

The pupil may or may not be conscious of the fact that he is learning how to study. That is not an important matter in the lower grades. However, it is more than likely that the interest of the pupil in his work would be increased by knowing the aim of the teacher, provided he has been shown the value of having the ability to study—a power enabling him to solve problems out of, as well as within, the classroom. While the pupil may not know the full purpose in what he does, the teacher must be very clear in the matter. She must also be familiar with the way, so as to direct the efforts of the pupil effectively towards the mastery of study. The pupil will, later of course, have a full knowledge of the importance of what he has done, even if he does not know now. If the pupil is not given the proper aid by the teacher, he may succeed by accident, but he will rarely succeed so well that he might not have done better with proper help. If the teacher is not skillful in giving assistance there may be great waste of the pupil's time even though he is fairly successful; but, more serious than this there is the likelihood of developing improper and wasteful habits of study which will be remedied with great difficulty, or probably not at all, and so discount the pupil's powers and possibilities for his entire life.

Professor F. M. McMurry, in *"How to Study,"* says: "Bad methods of study easily become a serious factor in adult life, acting as a great barrier to one's growth and general usefulness."

A class with a teacher who does not know how to study is only a caricature of what a class ought to be. Whatever else a scheme for education may have, if it has teachers who lack this knowledge, there is no such thing as education, in a real sense, possible. Failure to teach pupils how to study is a menace to our national welfare, it is a crime against humanity.

Those matters which affect the health and physical conditions of pupils are of very great importance and should receive attention first. It frequently happens that class sluggishness is due to foul air. Indeed, pupils need instruction in the importance of fresh air so that they may properly attend to the matter at home. The lighting should be carefully arranged so as to avoid dim lights, cross lights, glaring lights, etc. The class and study rooms should be made neat and attractive. Slovenly and uncouth surroundings will eventually result in corresponding mental habits. Untidy environment is likely to be morally harmful, leading to lower standards of living. The pupil should be aided in providing himself with such associations as will cultivate a cheerful, hopeful state of mind.

The teacher should be quick to observe physical defects in his pupils. The writer has a number of cases in mind in which the retardation of pupils was entirely due to defects in hearing or in vision, yet the pupils had been in school a number of years before these defects were observed. By proper attention such defects may be removed, or materially helped by rearranging the seating of a class, etc.

The pupil must assume, not only the mental, but also the physical attitude of attention. It will aid in securing the proper mental effort to place the body in that condition in which the pupil finds it when he gets the best results from his study. If he loafs the mind will tend to assume a similar condition. All study must be under the power of concentration, with intense application, when the mental energy is at white heat, for it is work under such conditions that gives results of real value. It is under such conditions that new and worth-while ideas will, and have, come to men—no one knows just how nor whence. The world's greatest discoveries have been the result of intense concentration on the problem, living with it, going to the limit of human power, until the light suddenly, but mysteriously broke.

Professor Carmichael says: "From the ore of thought important truth can usually be extracted only under the heat of a glowing zeal, when the mind is surcharged with that determination which arised from strongly motivated activity. Work which proceeds not at white heat is coldly done and possesses not the fire of vitality. . . . One cannot successfully woo the science of mathematics except under the inspiration of high motives. She refuses to consort with sordid aims. She can be happy only with him of high ideals who cherishes her nobler qualities; and only to him will she yield her increase for the blessing of mankind."

The writer had a pupil in his geometry class recently, who had been weak in her work. Suddenly she surprised the class by giving a perfect recitation on a difficult piece of advanced work. In telling the class how she accomplished this result, she said that at first the problem seemed impossible to her, but that she said to herself that she could and must get it. With this determination she set to work and did not give up until she was sure of complete mastery. This sort of determination is necessary for the success of the student. However, he must not work under such pressure for long periods without change. Forty minutes of such work followed by twenty minutes of relaxation will give better results over a long period than uninterrupted plodding. The student should work without worry, fluster, or nervousness, and he should be constantly on the alert, as he studies, for improvements in his methods of work. This sort of work is impossible if one goes immediately to studying after eating. Then, too, it is not possible to go immediately from intense work in one subject to the same sort of work in another unrelated subject. To be convinced of this one has but to try the experiment. One might as well expect a train under high speed to reverse itself instantly. The machinery used by the mind seems to be subject to the same laws, as regards momentum, as prevail in the physical world. The mind must have time to readjust itself before it can follow effectively a new line of effort. This fact needs more attention in our teaching. It is frequently overlooked. We often expect the pupil to change immediately from mathematics to history, from history to physics, from physics to language, etc.; the result is, that the student gets the wrong notion of what study is. We defeat ourselves in trying to educate him according to this plan.

The student must have faith in his ability to do the work assigned, unless he has this he cannot apply himself in such a way as to get results; without this faith the battle is lost before it is begun, he does not expect success and is not likely to get it. The pupil ought to remember that the work assigned is within his power to do or it would not be required of him. If the work is impossible for the student to do the fault lies with the teacher.

The pupil must have enough information about the assigned work to convince him of its importance and to arouse his interest in it. The advanced work should be studied as soon as possible after the class in that subject, while the interest is strong, and while the facts are fresh in the mind. Before making a detailed study of the new work the student should make a general survey of the field to be covered. This should be followed by a review of the preceding work which may have a bearing on the new problem. If there seems to be a lack of interest he should try to ascertain the cause and proceed to remedy the defect. Interest may be aroused by recalling the values of the topic as far as known, by the desire not to be outclassed by other pupils, or by the satisfaction which one may anticipate in mastering a difficult problem, giving assurance of increased strength, and of ability to meet other difficult situations in life with a corresponding determination to succeed.

The student should pause at frequent intervals during his study of a lesson, for the purpose of recalling points of importance, for comparing these and relating them to other information. It is advisable to have a note book in which should be written a digest, or summary of the lesson in the pupil's own words, rather than in those of the book. The book should be freely underscored and marginal notes written if these seem of value for a rapid review or for comparison. Supplementary leaves may be pasted in the book at the proper place if there is use for them. An unabridged dictionary, encyclopedia, other texts, and all available information should be convenient for reference. These helps should be used when needed for reference. Nothing should be guessed at, a meaning of a word, a statement needing verification, a reference which is not fully understood, these should be looked up immediately. This is a part of real study, any different procedure would develop bad study habits. After the lesson has been studied carefully, the student should select the *most* important points of the lesson and put extra time in the further fixing of these in his mind; about these as centers he should relate the points of less importance. Nor is this enough, he should find, if possible, instances in his own experience where the lesson applies. Finally, the important points, definitions, choice statements should be memorized. But at this juncture memorizing is a simple matter. The important parts will usually fix themselves in the memory because they have been thoroughly mastered. This procedure followed by careful attention to the recitations and discussions of the class when the lesson is considered there, will almost guarantee success to the student. The process may seem laborious at first but when it becomes a habit it will take less time, and the pleasure and profit will give returns a thousand fold. Such a student will have developed self-reliance. He will appreciate the spirit of such men as Grant, who would not allow anyone to help him solve a problem in his mathematics.

If the lesson should be mathematics the pupil should form the habit of checking his work. This will not necessarily shake his confidence in himself, but it should lead him to habitually suspend judgment on an issue until he has weighed all available evidence in the case, and until he is reasonably sure of his conclusions. This is the sort of person whose judgment on important matters is eagerly sought by his fellows—he becomes one of the influential persons in his community.

If the pupil has failed with his lesson after doing the best he knows how, it is a good plan, occasionally, to have him tell the class his method of study. This may be handled in such a way by the teacher that he will be able to correct his methods of work, and others will get valuable suggestions; to say nothing of the valuable hints the teacher himself will get regarding the individuals of the class. However, the spirit of helpfulness must be maintained, in other words, the discussion must be constructive, and not add to the discouragement of the pupil. Valuable suggestions may also be had by having a successful pupil tell a class of his method of work, providing he can do it in the proper spirit.

Each pupil must be trained to use the teacher to the utmost. He should feel personal responsibility in seeking out the teacher to clear up obscure points in the lesson, especially lessons that have already been studied in class. Of course it should not be too easy to get help in this way; the pupil must learn to do his utmost first. Work with individuals should never replace the work which would be more valuable when given in class. Practically all teaching, whether it is the subject or how to study the subject, should be made a class affair. Teaching individuals is a waste of time and effort. This experiment was tried recently in the University High School of one of our State universities. After teaching a class in mathematics individually for two years, the teacher declared it a failure.

The preparation for examinations usually gives unnecessary worry to the student. A little instruction as to the purpose and value of examinations would aid greatly. Their chief value is not to determine what grade the pupil ought to get for his work in the subject; although, in case of doubt, they may be valuable in this respect. The teacher is usually sufficiently well informed as to the rank of the pupil before the examination is given. The examination has its chief value in the preparation the pupil makes, providing this is done in the proper manner. The examination gives opportunity to review the work covered during an extended period. The important points are selected and related to each other. In a sense, a bird's eye view is taken over a considerable portion of the work so that it can be seen in its larger and more important relations to other problems and to life in general. Just as the subject should begin with a general survey of the field to be covered, for the sake of unity, so the course should end with a review for the same purpose. This will impress upon the mind a consciousness of the distinct gain and achievements in the course—the real work that has been done and its value. In preparing for the examination the student should go over the entire field picking out the important points, and making a list of the points which he recalls with greatest difficulty. The next effort should be made upon this new list; after study attempt to recall the points; make a new list of the points which still give difficulty. Continue this process until all the points selected as of first importance are thoroughly well known. This process may be repeated, if necessary, and if there is time, for the points of secondary importance. However, if the examination is a fair one, that is if it does not contain unimportant topics and purely catch questions, the student should be well prepared after he has mastered the most important topics. His power of discrimination will have been developed while studying the course, providing both he and the teacher have been doing conscious work. The chief source of worry for the student results from neglect of his work during the term. There is no excuse for this and there is really no remedy. Neglect of work during the term defeats the whole purpose of the study whatever grade may be obtained. It is, unfortunately, possible for pupils so disposed to "bluff," and scheme their way through. This sort of thing is likely to result in disappointments later in life.

(*To be continued*)